

Close Wed: HW\_5A,5B,5C (7.1,7.2,7.3)

Office Hours: 1:30-3:00 in COM B-006

See new postings on website (you will want them for reference).

*Entry Task:* Evaluate

$$\int x^2 e^{x/2} dx$$

## 7.1 Integration by Parts (continued)

Summary:  $\int u dv = uv - \int v du$

1. Pick  $u = ??$ . The rest is  $dv$ .
2. Differentiate to get  $du$   
Integrate to get  $v$ .

Here are all the most common examples:

- a) Products:  $x e^x, x^2 \cos(3x), x \sin(5x)$
- b) Logarithms:  $\ln(x), x^{10} \ln(x), \dots$
- c) Inv. Trig:  $\sin^{-1}(x), \tan^{-1}(x), \dots$
- d) Products:  $e^x \sin(x), e^x \cos(x)$

*Example:*

$$\int \sin^{-1}(x)dx = \int \arcsin(x)dx$$

*Example:* (Never ending integration by parts and how to end it):

$$\int e^x \cos(x) dx$$

## 7.2 Trigonometric Integral Methods

*Goal:* A procedure to integrate any combination of trig functions.

*Basic motivating examples:*

**All of these below** could be done with substitution: What is  $u$ ?

$$\int \sin^4(x) \cos(x) dx$$

$$\int \sin(x) \cos^3(x) dx$$

$$\int \tan^5(x) \sec^2(x) dx$$

$$\int \sec^6(x) \sec(x) \tan(x) dx$$

*Idea:* Use trig identities to turn a problem into a substitution problem like above.

## Tools

Essential Trig Identities:

$$\tan(x) = \frac{\sin(x)}{\cos(x)}, \cot(x) = \frac{\cos(x)}{\sin(x)},$$
$$\sec(x) = \frac{1}{\cos(x)}, \csc(x) = \frac{1}{\sin(x)}.$$

$$\sin^2(x) + \cos^2(x) = 1$$

$$\tan^2(x) + 1 = \sec^2(x)$$

$$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$$

$$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$$

$$\sin(x) \cos(x) = \frac{1}{2} \sin(2x)$$

See my online posting (or Appendix D of your book) for a more general discussion of trig identities.

Add these to your table of integrals you know (an updated table is already on the website):

$$\int \tan(x) dx = \ln |\sec(x)| + C$$

$$\int \sec(x) dx = \ln |\sec(x) + \tan(x)| + C$$

$$\int \sec^3(x) dx =$$
$$\frac{1}{2} \sec(x) \tan(x) + \frac{1}{2} \ln |\sec(x) + \tan(x)| + C$$

**Case 1 (cosine or sine has an odd power)**

*i)*  $\int \sin^2(x) \cos^3(x) dx$

**Case 2 (Both have even powers)**

*i)*  $\int \cos^2(x) dx$

*ii)*  $\int \sin^3(x) dx$

*ii)*  $\int \sin^4(x) dx$

**Case 3 (even power on secant)**

$$\int \tan^2(x) \sec^4(x) dx$$

**Case 4 (Odd power on secant, and at least one tangent)**

$$\int \tan^3(x) \sec^5(x) dx$$

What is the first step in each integral below?

$$\int \sin^3(x)\cos^4(x)dx$$

$$\int \sin^5(x)\cos^3(x)dx$$

$$\int \cos^4(x)dx$$

$$\int \tan^5(x)\sec^4(x)dx$$

$$\int \tan^5(x)\sec(x)dx$$